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The remote Wigner polaron in a two-dimensional electron system

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Abstract. – The property of the remote electron a certain distance away from a Wigner crystallized two-dimensional electron system (2DES) is investigated. The electron mass of the remote electron is: 1) modified by the periodic potential of the Wigner crystal (WC), and 2) renormalized due to the WC phonons, *i.e.* the polaron effect. The latter is obtained using the Feynman path integral method which also provides us with the correlation energy. In this novel polaron problem the electron-phonon interaction can be controlled by the distance between the two 2DES.

A two-dimensional electron system (2DES) under appropriate conditions can crystallize into a hexagonal Wigner crystal [1]. Such a Wigner crystal (WC) is realized in the classical system of electrons above the surface of liquid helium [2] and in the quantum 2DES in heterojunctions in the presence of a large magnetic field [3]. Recently, the bilayer electron system (BLES) composed of two interacting 2DES separated a certain distance from each other has been investigated experimentally and theoretically. The BLES is realized in a high-quality double quantum well [4] or in a single wide quantum well [5]. The distance between the two 2DES introduces a new degree of freedom through which the electron-electron interaction can be modified. For example, the quantum Hall effect is modified by the inter-layer interaction [4], [5] and different WC phases are predicted such as square or rhombic lattices which are not stable in single layer systems [6].

In previous studies on BLES, the densities of the two 2DES are assumed to be equal. Here, we investigate a novel case where the BLES is composed of a very dilute 2DES and in which the other 2DES is crystallized into a WC. An alternative system would be a double-barrier system as used in resonant-tunneling experiments in which the quantum well contains a WC and there is a single electron propagating outside one of the barriers. In fig. 1 (a) a schematic diagram of the electron density profile of the system is shown. We assume that the potential barrier between the WC and the remote electron is sufficiently wide such that tunneling and

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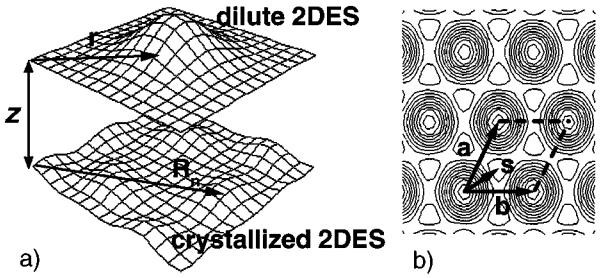

Fig. 1. – (a) Schematic diagram of the electron density profile of the WC and the remote electron. (b) Contour plot of the periodic potential due to the crystallized 2DES as felt by the remote electron. The Bravais cell of the hexagonal lattice is enclosed by the thick dashed lines.

exchange between them can be neglected. In the dilute 2DES, the electron density is assumed to be so small that the intra-layer electron-electron interaction is negligible and a one-particle picture is appropriate. The remote electron feels the periodic potential of the crystallized 2DES leading to a new electron band structure which, *e.g.*, results into an effective mass, which is heavier than the bare-electron mass in the host material. Moreover, this electron distorts the WC locally and the composite quasi-particle: electron + WC distortion is called a *Wigner polaron*. This WC distortion is described in terms of virtual phonons of the WC. The resulting electron-phonon interaction in BLES has the remarkable feature that its strength can be modified by changing the distance of the two 2DES.

In fig. 1 (b) we show the contour plot of the periodic potential due to the crystallized 2DES as felt by the remote electron. The Bravais cell of the hexagonal lattice is enclosed by the dashed line, the vectors $\mathbf{a}$ and $\mathbf{b}$ are the basis of the lattice. In order to minimize the Coulomb repulsion, the remote electron will be situated at the two triangle centers in the Bravais cell, which are located at $\mathbf{s} = (\mathbf{a} + \mathbf{b})/3$ and $2\mathbf{s}$.

The remote electron will be in a Bloch state and has a sub-band structure due to the presence of the periodic potential of the crystallized 2DES. To obtain the effective electron mass as a function of the layer distance, we employ the variational method and use a Gaussian-type Wannier basis [7]. The electron Bloch state $\psi_{\mathbf{k}}$ is taken as follows:

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{n}} \sum_{j=1,2} e^{-i\mathbf{k}\cdot(\mathbf{R_n}+j\mathbf{s})} w(\mathbf{r} - \mathbf{R_n} - j\mathbf{s}) \,, \tag{1}$$

where

$$w(\mathbf{r}) = \frac{1}{\sqrt{2}\mu} e^{-\mathbf{r}^2/2\mu^2} \,. \tag{2}$$

Here, μ is a parameter which measures the extent of the remote electron in the plane, $\mathbf{k}$ is a 2D wave vector in the first Brillouin zone, $\mathbf{r}$ is the 2D electron position coordinate in the dilute 2DES, $\mathbf{R_n}$ is the 2D hexagonal lattice vector which is given by $n_x\mathbf{a} + n_y\mathbf{b}$, with $\mathbf{n} = (n_x, n_y)$ integers, and $w(\mathbf{r})$ is the Gaussian-type Wannier basis. In the Bravais cell of the hexagonal lattice there are two points, *i.e.* at $\mathbf{s}$ and $2\mathbf{s}$, where the WC potential is minimum. In the vicinity of these points, the potential is nearly quadratic and therefore the Wannier state is well described by the Gaussian function as in (2). The variational parameter μ is determined by minimizing the energy

$$E_{\mathrm{WC}}(\mathbf{k}) = \left\langle \psi_{\mathbf{k}} \left| \left\{ \frac{\mathbf{p}^2}{2m} + U \right\} \right| \psi_{\mathbf{k}} \right\rangle \Big/ \langle \psi_{\mathbf{k}} \mid \psi_{\mathbf{k}} \rangle \,, \tag{3}$$

where m is the electron mass in the host material, $\mathbf{p}$ is the momentum of the 2D electron and $U(\mathbf{r})$ is the periodic potential of the crystallized 2DES,

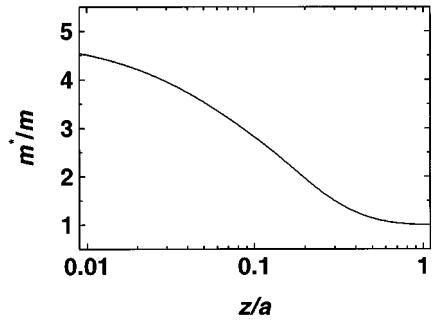

Fig. 2. – The z-dependence of the effective mass m^* in the static periodic potential of the WC for $n_{\rm s} = 10^7$ cm^{-2} which implies $a = 3.40$ μm. Here, m is the bare-electron mass in the host material, and a is the lattice constant of the WC.

$$U(\mathbf{r}) = \frac{e^2}{\varepsilon} \sum_{\mathbf{n}} \frac{1}{\sqrt{(\mathbf{r} - \mathbf{R_n})^2 + z^2}}\,, \tag{4}$$

where $-e$ is the electron charge, ε the dielectric constant of the host material, and z the distance between the WC and the remote electron. The extent of the 2DES in the z-direction is taken to be zero for convenience. Calculating the integral (3) in wave-number space, we obtain the following expression:

$$E_{\rm WC} = \frac{1}{2}\left\{\mathbf{k}^2 + C_{\mathbf{k}}^{-1} \sum_{\mathbf{K}\neq 0} (\mathbf{k}^2 + \mathbf{k}\cdot\mathbf{K}) e^{-\mu^2(\mathbf{k}+\mathbf{K})^2} \cos^2\left(\frac{1}{2}\mathbf{K}\cdot\mathbf{s}\right)\right\} +$$

$$+ C_{\mathbf{k}}^{-1} \frac{2\pi n_{\rm s}}{a_{\rm B}} \sum_{\mathbf{K}\neq 0} \frac{e^{-Kz}}{K} D_{\mathbf{k}}(\mathbf{K})\,, \tag{5}$$

where $\mathbf{K}$ is a 2D reciprocal lattice vector of the hexagonal WC, $a_{\rm B} = \hbar\varepsilon/me^2$ is the effective Bohr radius, $n_{\rm s}$ is the 2D electron density,

$$C_{\mathbf{k}} = \sum_{\mathbf{K}\neq 0} e^{-\mu^2(\mathbf{k}+\mathbf{K})^2} \cos^2\left(\frac{1}{2}\mathbf{K}\cdot\mathbf{s}\right), \tag{6a}$$

and

$$D_{\mathbf{k}}(\mathbf{G}) = \sum_{\mathbf{K}} \exp\left[-\frac{1}{2}\mu^2\{(\mathbf{k}+\mathbf{K}+\mathbf{G})^2 + (\mathbf{k}+\mathbf{K})^2\}\right] \cos\left[\frac{1}{2}(\mathbf{G}+\mathbf{K})\cdot\mathbf{s}\right] \cos\left(\frac{1}{2}\mathbf{K}\cdot\mathbf{s}\right). \tag{6b}$$

Here we use unit such that $\hbar = m = K_1 = 1$ ($K_1 = 4\pi/\sqrt{3}a$ is our unit for the reciprocal lattice vector where a is the lattice constant of the WC).

Expression (5) has the parabolic form $k^2/2m^*$ for small k, where m^* is the effective mass of the Bloch state. In fig. 2 we show the z-dependence of the effective mass m^* of the remote electron when it moves in the potential of the static WC. We took the material constants of GaAs: $\varepsilon = 12.53$ and $m = 0.067m_0$ and considered the electron density $n_{\rm s} = 10^7$ cm^{-2} which results into $a = 3.40 \times 10^3$ nm. When the distance z between the two 2DES becomes smaller than the lattice constant a, the effective mass increases rapidly due to the strong potential of the crystallized 2DES.

In a second stage we include the deformation of the lattice due to the presence of the remote electron which is a distance z away. Such an electron will locally polarize the WC which can be viewed as the excitation of virtual phonons of the WC. In the above analysis we assumed

the WC crystal to be frozen and the electrons were located in fixed lattice positions. The deformation interaction energy of such a remote electron with the WC is given by

$$H_{\rm int} = \frac{e^2}{\varepsilon} \sum_{\mathbf{n}} \left\{ \frac{1}{\sqrt{(\mathbf{r} - \mathbf{R_n} - \mathbf{u}(\mathbf{R_n}))^2 + z^2}} - \frac{1}{\sqrt{(\mathbf{r} - \mathbf{R_n})^2 + z^2}} \right\} . \tag{7}$$

Here, $\mathbf{r}$ is the 2D position of the remote electron, and $\mathbf{u}(\mathbf{R_n})$ is the displacement of the WC electron bound at the hexagonal lattice side $\mathbf{R_n}$. If we linearize eq. (7) in $\mathbf{u}(\mathbf{R_n})$ and take the long wavelength limit, the electron interacts with longitudinal WC phonons. The interaction energy can now be written as follows:

$$H_{\rm int} = \sum_{\mathbf{k}} \frac{V_{\mathbf{k}}}{\sqrt{\Omega}} e^{-i\mathbf{k}\cdot\mathbf{r}} (a_{\mathbf{k}} + a^{\dagger}_{-\mathbf{k}}) . \tag{8}$$

Here, Ω is the system area, $a_{\mathbf{k}}$ and $a^{\dagger}_{\mathbf{k}}$ are the annihilation and creation operator of the longitudinal WC phonon with wave vector $\mathbf{k}$ and frequency $\omega_{\mathbf{k}}$. The interaction strength $V_{\mathbf{k}}$ is given by

$$V_{\mathbf{k}} = \frac{2\pi}{a_{\rm B}} \sqrt{\frac{n_{\rm s}}{2\omega_{\mathbf{k}}}} e^{-kz} , \tag{9}$$

in units $\hbar = m = K_1 = 1$, where $\omega_{\mathbf{k}}$ is the dispersion relation of the longitudinal WC phonon which is given by L. Bonsall and A. A. Maradudin [8],

$$\omega_{\mathbf{k}} = \left\{ \frac{2\pi n_{\rm s}}{a_{\rm B}} (k - 1.3167k^2) \right\}^{1/2} . \tag{10}$$

Thus the Hamiltonian H for the *Wigner polaron* interacting with the longitudinal WC phonons becomes

$$H = \frac{\mathbf{p}^2}{2m^*} + \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(a^{\dagger}_{\mathbf{k}} a_{\mathbf{k}} + \frac{1}{2} \right) + H_{\rm int} . \tag{11}$$

The first term is the kinetic energy of a single electron with 2D momentum $\mathbf{p}$ and the effective mass m^* whose z-dependence is shown in fig. 2. The second term is the Hamiltonian of the longitudinal WC phonon. Equation (11) is a standard polaron Hamiltonian which we solve using the path integral representation of the partition function and introducing the Feynman trial action [9]. We obtain the following expression for the Wigner polaron energy at zero temperature:

$$E = \frac{(v-w)^2}{2v} - \int_0^{\infty} {\rm d}\tau \frac{1}{2\pi} \int_0^{k_{\rm c}} {\rm d}k \, k \mid V_{\mathbf{k}} \mid^2 \exp\left[\frac{-\mathbf{k}^2}{m^*} f(\tau) - \omega_{\mathbf{k}} \mid \tau \mid \right] , \tag{12}$$

where $k_{\rm c}$ is the cutoff parameter determined by the first Brillouin zone boundary, which equals $1/2$ in our units, and

$$F(\tau) = \frac{w^2}{2v^2} \mid \tau \mid + \frac{v^2 - w^2}{2v^3} (1 - e^{-|\tau| v}) , \tag{13}$$

where v and w are the variational parameters of the Feynman trial action [9].

In fig. 3 (a), we show the z-dependence of the polaron energy $-E$ in units of $E_0 = \hbar^2 K_1^2/m$ ($E_0 = 5.19$ μeV for GaAs with $n_{\rm s} = 10^7$ cm^{-2}). This energy is equivalent to the Coulombic correlation energy between the remote electron and the electrons in the WC. When the distance z decreases, the energy $-E$ increases because the electron interacts more strongly with the WC phonon resulting in a large distortion of the WC lattice. For reference we show the energy when $m^*/m = 1$ (dashed curve), *i.e.* no band effects due to the static WC. The polaron model

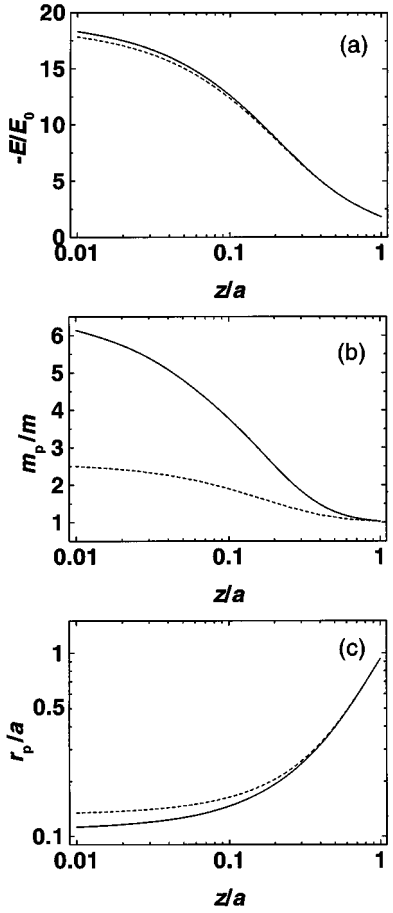

Fig. 3

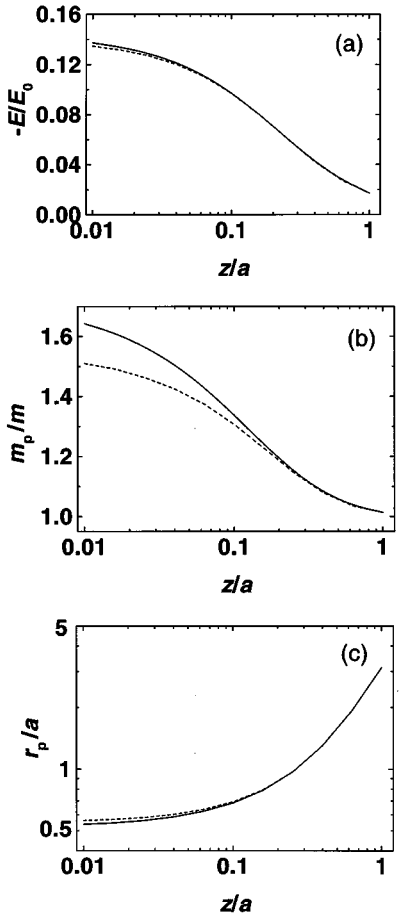

Fig. 4

Fig. 3. – The z-dependence of (a) the polaron energy $-E$, (b) the polaron mass $m_{\rm p}$, and (c) the polaron radius $r_{\rm p}$. The energy is in units of $E_0 = \hbar^2 K_1^2/m = 5.19$ μeV for $n_{\rm s} = 10^7$ cm^{-2} and $a = 3.40$ μm is the lattice constant of the hexagonal WC.

Fig. 4. – The same results as in fig. 3 but now for a higher-density sample of $n_{\rm s} = 10^{11}$ cm^{-2} which gives for the energy unit $E_0 = 51.9$ meV and $a = 34.0$ nm.

mass $m_{\rm p}$ is defined by $m^* v^2/w^2$ and this value is related to the mass enhancement induced by the electron-phonon interaction. In fig. 3 (b), the polaron mass $m_{\rm p}$ scaled by the bare mass m is plotted as a solid curve. The dashed curve is the result when we assumed $m^*/m = 1$. Notice that there is a substantial mass renormalization when the electron is close to the WC. This mass enhancement increases when we include band effects due to the static WC. The polaron radius $r_{\rm p}$ defined by $\{v/m^*(v^2 - w^2)\}^{1/2}$ is plotted in fig. 3 (c) by the solid curve and the dashed curve is the result when $m^*/m = 1$. As the distance z becomes small and the electron interacts with the phonon much stronger, the polaron radius shrinks continuously. This is in contrast to the acoustic polaron systems in which a discontinuous shrinkage is possible, depending on the value of the cutoff parameter, due to a transition from the free to the self-trapped state [10].

Similar results for a higher-density system, *i.e.* $n_{\rm s} = 10^{11}$ cm^{-2} are shown in fig. 4. Now $E_0 = 51.9$ meV and $a = 34.0$ nm. In this case the effective mass m^* can be calculated from the

formula $m^* = 1 + 24(2\pi n_s/a_B)^2 \exp[-2z]$ which is obtained by a $\mathbf{k}\cdot\mathbf{p}$ perturbation method. The mass renomalization is now substantially smaller than in the low-density case, *i.e.* a factor 3.5. The energy shift is in relative units much smaller but in absolute values it is a factor of 100 larger for small z, namely $-E \approx 10$ meV for $n_s = 10^{11}$ cm^{-2} and $-E \approx 0.1$ meV for $n_s = 10^7$ cm^{-2}.

The electron mass is a basic quantity in transport phenomena and in cyclotron resonance. The predicted mass and energy enhancement should be observable experimentally. The deformation energy is related to the Coulomb (quasi-)gaps seen in a tunneling experiment of a single electron into a Wigner crystallized 2DES [11], [12]. Recently, an energy shift in the position of the tunneling peak was observed in ref. [12] when a large perpendicular magnetic field was applied. In such a case the 2DES is strongly correlated, although not yet in the Wigner crystal phase. An experimental shift in the tunneling peak position of about 2 meV for $n_s = (0.7\text{–}3) \times 10^{11}$ cm^{-2} is comparable in size with our theoretical result for the polaron energy of $-E = 0.04\ E_0 = 2.1$ meV for $z/a = 0.4$ and $n_s = 10^{11}$ cm^{-2}. We took $z = 0.4a = 13.6$ nm $\approx (11 + 5.8/2)$ nm as the distance between the 2DES and the tunneling electron, *i.e.* the tunneling barrier in the experiment of ref. [12], had a width of 11 nm and the width of the quantum well the remote electron was residing in was 5.8 nm.

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