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The remote plasmon polaron

H. Kato[1](*), F. M. Peeters[1](**) and S. E. Ulloa[2](***)

[1] *University of Antwerp (UIA), Department of Physics, Universiteitsplein 1 B-2610 Antwerpen, Belgium*

[2] *Department of Physics and Astronomy, Condensed Matter and Surface Sciences Program Ohio University, Athens - Ohio 45701-2979, USA*

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Abstract. – The Coulomb correlation of a remote electron separated from a two-dimensional electron system (2DES) is studied in the presence of a magnetic field applied perpendicular to the 2DES. The polarization produced by the remote electron is described as the excitation of virtual plasmons in the 2DES. The resulting composite quasi-particle, electron + polarization, is therefore called a *plasmon polaron*. The ground state energy of this quasi-particle is calculated using second-order perturbation theory and the results are related to recent resonant tunneling experiments. Qualitative agreement of the shift in the resonant tunneling peak with magnetic field is obtained.

The electronic Coulomb interaction in a doped semiconductor system leads to exchange and correlation contributions to the electron energy similar to those in metals [1, 2]. It has been recently shown that resonant tunneling can reveal this Coulomb interaction as a shift in the bias position of the tunneling peak. A quantum resonance device was used, consisting of a double barrier and quantum well containing a single two-dimensional electron gas (2DEG) as injector into an "empty" quantum well region [3]. The dependence of the peak position on magnetic field and temperature were studied. A tunneling electron can be regarded in such experiments as a remote electron which is separated from the two-dimensional electron system (2DES) and interacts via the Coulomb interaction with the 2DES left behind. This interaction energy is related to the Coulomb (quasi-) gap obtained in the resonant tunneling experiment of a single electron into a 2DES, and between interacting adjacent 2DESs [3-5]. This general problem has attracted significant theoretical interest, and is well described in high or low field ranges by fully quantum-mechanical or semiclassical descriptions, as appropriate [6-8]. Here

(*) Permanent address: Hakodate National College of Technology, 14-1 Tokura-cho Hakodate, 042 Hokkaido, Japan. E-mail: kato@cc.hakodate-ct.ac.jp
(**) E-mail: peeters@uia.ua.ac.be
(***) E-mail: sulloa1@ohiou.edu

we present an alternative theoretical approach which covers the entire magnetic-field range, and provides an intuitive explanation of the observed tunneling peak voltage shifts in the experiments of Lok *et al.* [3].

In a previous study, we formulated the problem of a remote electron interacting with the 2DES as a polaron problem, where the 2DES was assumed to be in the (low-density) Wigner crystal state. The electron polarizes the 2DES, and in that density regime this is treated as the excitation of phonons in the 2DES within a discrete phonon model [9]. In that formulation we neglected magnetic fields and the corresponding Landau levels. Using a previous treatment of the electronic state in a metal [2] and the dielectric function of a 2DES [10, 11], we extend here our polaron model to arbitrary magnetic fields and to the 2DES liquid state.

When a Landau level is fully filled, electrons in that level cannot respond to an external perturbation. Thus, only electrons in partially filled Landau levels are responsible for any local polarization by an external potential. The dielectric function exhibits oscillations reflecting the occupation of Landau levels and the plasmon frequency is modified by this dielectric function. As a consequence, the strength of the Coulomb interaction between the remote electron and the 2DES will be an oscillating function of the magnetic field. This interaction is calculated here within second-order perturbation theory.

A remote electron placed a distance d away from the 2DES, which lies on the xy-plane, induces a modulation of the election density $n(\vec{r}\,)$ of the 2DES. If we denote the position of the j-th electron in the 2DES by $\vec{r}_j = (x_j, y_j)$, the electron density is expressed as

$$n(\vec{r}\,) = \sum_{j=1}^{N} \delta(\vec{r} - \vec{r}_j) = \frac{1}{\Omega} \sum_{\vec{k}} n_{\vec{k}} \exp[i\vec{k}\cdot\vec{r}\,]\,, \tag{1}$$

with $\vec{r} = (x, y)$ being the 2D position, N the total number of electrons, Ω the system area, and $n_{\vec{k}}$ the Fourier coefficient with 2D wave number $\vec{k}$. We will describe the density fluctuation in terms of collective excitations of the system (magnetoplasmons). When these excitations are formulated quantum-mechanically, the coefficient $n_{\vec{k}}$ is expressed via annihilation and creation operators for each magnetoplasmon $a_{\vec{k}}$ and $a^{\dagger}_{\vec{k}}$ as follows: $n_{\vec{k}} = \lambda_{\vec{k}}(a_{\vec{k}} + a^{\dagger}_{-\vec{k}})$, where the coefficient $\lambda_{\vec{k}}$ is to be determined later [2]. Then, the Hamiltonian of the remote electron in a magnetic field interacting with these collective excitations becomes

$$H = \frac{1}{2m}\left(\vec{p} + \frac{e}{c}\vec{A}(\vec{r}\,)\right)^2 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} a^{\dagger}_{\vec{k}} a_{\vec{k}} + H_{\rm int}\,. \tag{2}$$

Here, the first term refers to the remote electron, and the vector potential $\vec{A}(\vec{r}\,)$ is chosen in the symmetric gauge to be $\vec{A}(\vec{r}\,) = \frac{1}{2}B(-y, x, 0)$, with B the magnetic field along the z-axis. The frequency $\omega_{\vec{k}}$ is related to the plasmon and will be derived later.

The interaction Hamiltonian $H_{\rm int}$ is determined by the potential $U_{\rm in}(\vec{r}, z)$ induced by the remote electron and given by Poisson's equation:

$$(\nabla^2_{\rm 2D} + \partial^2_z) U_{\rm in}(\vec{r}, z) = -\frac{4\pi e}{\epsilon_0}\{n(\vec{r}\,) - n_{\rm s}\}\delta(z), \tag{3}$$

where $\nabla^2_{\rm 2D}$ is the 2D Laplacian, ϵ_0 is the dielectric constant of the host material, and $n_{\rm s} = N/\Omega$ is the average electron density of the 2DES. The solution of Poisson's equation is given by

$$U_{\rm in}(\vec{r}, z) = -\frac{1}{\Omega}\sum_{\vec{k}} \frac{2\pi e}{\epsilon_0 k} \exp[-k|z|\,]\, n_{\vec{k}} \exp[-i\vec{k}\cdot\vec{r}\,]\,, \tag{4}$$

with $k = |\vec{k}|$. The interaction Hamiltonian with the remote electron at $z = d$, $H_{\rm int} = -eU_{\rm in}(\vec{r}, z = d)$ becomes then

$$H_{\rm int} = \sum_{\vec{k}} \frac{V_{\vec{k}}}{\sqrt{\Omega}} \exp[-i\vec{k}\cdot\vec{r}]\,(a_{\vec{k}} + a^{\dagger}_{-\vec{k}})\,, \tag{5}$$

where we defined the interaction strength $V_{\vec{k}}$ as

$$V_{\vec{k}} = \frac{2\pi e^2}{\sqrt{\Omega}\epsilon_0 k}\lambda_{\vec{k}} \exp[-kd\,]\,. \tag{6}$$

Next, we determine the coefficient $\lambda_{\vec{k}}$ and the frequency $\omega_{\vec{k}}$ using the *f*-sum rule [2]. The 2D electrons in the 2DES in the presence of a magnetic field are described by the Hamiltonian

$$H_{\rm 2DES} = \sum_{j=1}^{N} \frac{1}{2m}\left(\vec{p}_j + \frac{e}{c}\vec{A}(\vec{r}_j)\right)^2 + \sum_{j\neq i}^{N} \frac{e^2}{2\epsilon_0|\vec{r}_i - \vec{r}_j|}\,. \tag{7}$$

The *f*-sum rule is obtained by evaluating the average $\langle E_n|[n_{-\vec{k}},[n_{\vec{k}}, H_{\rm 2DES}]]|E_0\rangle$, where $|E_0\rangle$ is the ground state, and $|E_n\rangle$ is the excited state of $H_{\rm 2DES}$ [1]. The *f*-sum rule was originally derived without a magnetic field, however, the resulting expression remains the same even if a magnetic field is applied. This *f*-sum rule is then

$$\sum_n \hbar\omega_{n0}|\langle E_n|n_{\vec{k}}|E_0\rangle|^2 = N\frac{\hbar^2 k^2}{2m}\,, \tag{8}$$

where $\hbar\omega_{\rm n0} = E_{\rm n} - E_0$. Within the plasmon-pole there is only one collective excitation for each wave vector $\vec{k}$, and we can put $\omega_{n0} = \omega_{\vec{k}}$ which reduces the sum rule to

$$\hbar\omega_{\vec{k}}\lambda^2_{\vec{k}} = N\frac{(\hbar k)^2}{2m}\,. \tag{9}$$

Substituting this expression into (6), we obtain

$$V_{\vec{k}} = \frac{2\pi e^2}{\epsilon_0}\sqrt{\frac{\hbar n_{\rm s}}{2m\omega_{\vec{k}}}} \exp[-kd\,]\,. \tag{10}$$

The interaction potential is given in terms of $\omega_{\vec{k}}$, and we proceed now to evaluate this excitation frequency.

The electrostatic potential $U_{\rm ex}(\vec{r}, z)$ produced by the remote electron located at $z = d$ similarly obeys Poisson's equation, such that the solution is given by

$$U_{\rm ex}(\vec{r}, z) = \frac{1}{\Omega}\sum_{\vec{k}} U^{\rm ex}_{\vec{k}}(z) \exp[i\vec{k}\cdot\vec{r}\,]\,, \tag{11}$$

with

$$U^{\rm ex}_{\vec{k}}(z) = \frac{2\pi e}{\epsilon_0 k} \exp[-k|d - z|]\,. \tag{12}$$

In the self-consistent field approximation, the total field (potential) in the 2DES at $z = 0$, $U^{\rm total}_{\vec{k}}(z = 0)$ is composed of the external potential $U^{\rm ex}_{\vec{k}}(0)$ and the induced potential $U^{\rm in}_{\vec{k}}(0)$, such that

$$U^{\rm total}_{\vec{k}}(0) = U^{\rm ex}_{\vec{k}}(0) + U^{\rm in}_{\vec{k}}(0) = \frac{U^{\rm ex}_{\vec{k}}(0)}{\epsilon(k)}\,, \tag{13}$$

where $\epsilon(\vec{k})$ is the dielectric function. The induced potential at points away from the plane is then given by

$$U^{\mathrm{in}}_{\vec{k}}(z) = -\exp[-k|z|]\frac{\epsilon(k)-1}{\epsilon(k)}U^{\mathrm{ex}}_{\vec{k}}(0)\,, \tag{14}$$

where $\exp[-k|z|]$ represents the propagator which transfers the potential at $z = 0$ to arbitrary z, as seen in eq. (4). In classical electrostatics the interaction energy between a test charge placed at $(\vec{r}, z) = (0, d)$ and the induced polarization is given by $W_{\mathrm{cl}} = (-e/2)U_{\mathrm{in}}(\vec{r} = 0, z = d)$. Using eqs. (12) and (14), we obtain

$$W_{\mathrm{cl}} = \sum_{\vec{k}} \frac{\pi e^2\{\epsilon(k)-1\}}{\Omega\epsilon_0 k\epsilon(k)}\exp[-2kd]. \tag{15}$$

On the other hand, if the test charge at $(\vec{r}, z) = (0, d)$ is described by the Hamiltonian (2), second-order perturbation theory gives the energy $W_{\mathrm{qm}} = \sum|V_{\vec{k}}|^2/\Omega\hbar\omega_{\vec{k}}$. Comparing the two energies W_{cl} and W_{qm}, we obtain

$$\omega^2_{\vec{k}} = \frac{2\pi e^2 n_{\mathrm{s}}}{\epsilon_0 m}k\frac{\epsilon(k)}{\epsilon(k)-1}\,, \tag{16}$$

for the effective excitation spectrum of the 2DES [2]. The first part, $\omega^0_{\vec{k}} = (2\pi e^2 n_{\mathrm{s}} k/\epsilon_0 m)^{1/2}$, corresponds to the classical $(k \approx 0)$ 2D plasmon energy, while the factor $\epsilon(\vec{k})/[\epsilon(\vec{k}) - 1]$ represents the modulation associated with the finite-k excitations of the electron gas.

What remains in achieving a complete description of H_{int} is securing a reliable description of $\epsilon(k)$, the dielectric function of the 2DES in a magnetic field. In linear response theory, a simple description of the dielectric function in a magnetic field which gives a general good description of the system is given by $\epsilon(k) = 1 + q_{\mathrm{s}}/k$, with the screening constant q_{s} given by [10]

$$q_{\mathrm{s}} = \frac{2\pi\hbar^2}{a_{\mathrm{B}}m}D(E_{\mathrm{F}}). \tag{17}$$

Here, $a_{\mathrm{B}} = \hbar^2\epsilon_0/me^2$ is the effective Bohr radius and $D(E)$ is the electronic density of states

$$D(E) = \sum_{n=0}^{\infty}\sum_{\alpha=\pm}\frac{1}{2\pi l^2_{\mathrm{B}}}\,\frac{1}{\Gamma\sqrt{\pi}}\,e^{-(E-E_{n\alpha})^2/\Gamma^2}\,, \tag{18}$$

where we have used Gaussian broadening of the Landau levels. For short-range impurity scattering the magnetic-field dependence of the Landau level width is given by $\Gamma = \Gamma_0 B^{1/2}$, where Γ_0 is a constant determined by the mobility of the 2DES [10,11]. In eq. (18) we use the magnetic length $l_{\mathrm{B}} = (\hbar/m\omega_{\mathrm{c}})^{1/2}$ and the Landau level energy $E_{n\alpha} = \hbar\omega_{\mathrm{c}}(n+1/2)+\alpha g\mu_{\mathrm{B}}B/2$, with the cyclotron frequency $\omega_{\mathrm{c}} = eB/mc$, the effective g-factor, $\alpha = +1$ (–1) for the up (down) spin state, the Bohr magneton $\mu_{\mathrm{B}} = e\hbar/2m_0c$, and m_0 is the electron bare mass. The Fermi level E_{F} is determined from the condition for the total number of electrons, so that at zero temperature

$$n_{\mathrm{s}} = \int_{-\infty}^{E_{\mathrm{F}}}\mathrm{d}E\,D(E)\,. \tag{19}$$

In fig. 1 the screening constant q_{s} and the Fermi energy E_{F} are shown as a function of the magnetic field B for a 2DES with electron density $n_{\mathrm{s}} = 3\times10^{11}\mathrm{cm}^{-2}$ and $\Gamma_0 = 1.5\ \mathrm{meV/T}^{1/2}$.

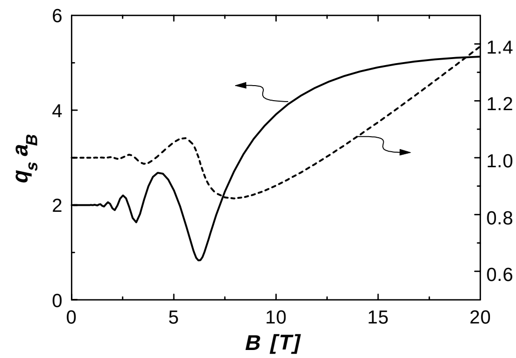

Fig. 1

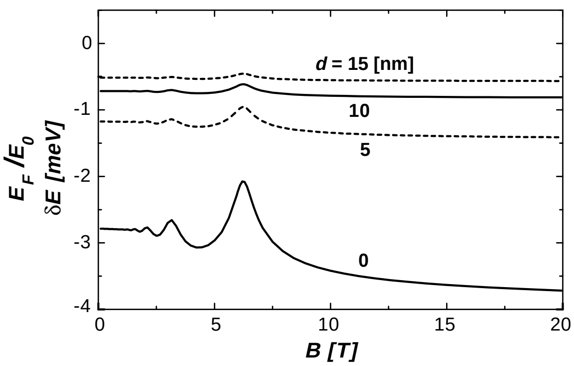

Fig. 2

Fig. 1. – The magnetic-field dependence of the screening constant q_s (solid curve) and the Fermi energy E_F (dashed curve) for a GaAs heterojunction with electron density $n_\mathrm{s} = 3 \times 10^{11}\mathrm{cm}^{-2}$ and Landau level width $\Gamma_0 = 1.5$ meV/T$^{1/2}$. The units of energy and length are $E_0 = 10.7$ meV and $a_\mathrm{B} = 9.90$ nm, respectively.

Fig. 2. – The magnetic-field dependence of the polaron energy for various d. The peaks in each curve occur at complete filling of the highest Landau level. The material parameters are the same as in fig. 1.

The units of energy and length are $E_0 = \hbar^2 \pi n_\mathrm{s}/m = 10.7$ meV and $a_\mathrm{B} = 9.90$ nm, respectively, for GaAs; $g = 0.52$, $\epsilon_0 = 12.5$, and $m/m_0 = 0.067$. When the highest Landau level is fully filled, the Fermi energy E_F changes abruptly and the screening constant q_s reaches a minimum, so that few electrons can shield the external field. In the limit of zero magnetic field, the energy unit E_0 equals the Fermi energy E_F and the screening constant (17) reduces to $q_\mathrm{s} = 2/a_\mathrm{B}$, the well-known 2D Thomas-Fermi constant. For simplicity we have neglected the finite extent of the 2DES along the z-axis.

At this point, we have a full description of the interaction term between the remote electron and the 2DES, H_int. Next we calculate the ground state energy of the remote electron interacting with the 2DES using second-order perturbation theory in the Hamiltonian (2), and further assuming that the remote plasmon polaron state has translational invariance. The ground state energy of the remote electron $\hbar\omega_\mathrm{c}/2$ (minus spin) is shifted in energy δE due to the interaction with the plasmons of the 2DES [12]:

$$E = \frac{1}{2}\hbar\omega_\mathrm{c} + \delta E, \tag{20}$$

with

$$\delta E = -\frac{1}{\Omega}\sum_{\vec{k}} \frac{|V_{\vec{k}}|^2}{\hbar\omega_{\vec{k}}} \int_0^\infty \mathrm{d}\tau \exp[-\tau + k^2 D_\mathrm{B}(\tau\omega_\mathrm{c}/\omega_{\vec{k}})], \tag{21}$$

and

$$D_\mathrm{B}(\xi) = \frac{{l_\mathrm{B}}^2}{2}\{1 - \exp[-\xi]\}. \tag{22}$$

In fig. 2, this polaron energy correction δE is shown as a function of the magnetic field B for different values of d, the distance of the remote electron to the 2DES, for the same material constants used in fig. 1. Each curve oscillates with magnetic field due to the changing

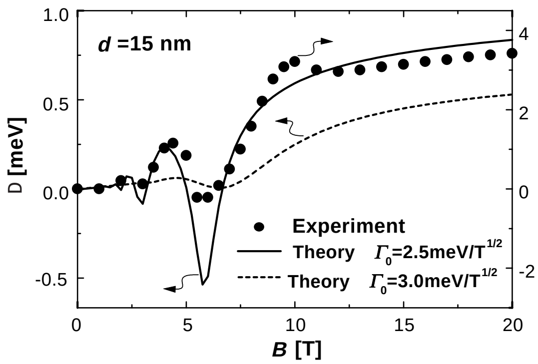

Fig. 3

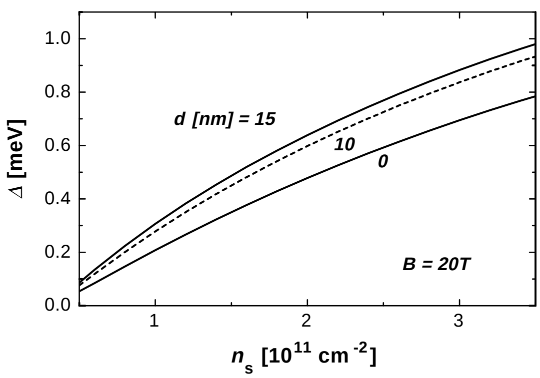

Fig. 4

Fig. 3. – The energy shift due to a magnetic field, $\Delta = W(d,B) - W(d,B=0)$, as calculated from the data in fig. 2. Solid and dash curves are shown for $\Gamma_0 = 2.5$ and 3.0 meV/T$^{1/2}$, respectively. Solid circles are the experimental data of ref. [3].

Fig. 4. – The energy shift Δ as a function of the electron density n_s for $B = 20$ T, and various d values.

occupation of the Landau levels which modulates the electron-plasmon interaction. It reaches a maximum when the highest Landau level is fully filled. This oscillation is directly related to the oscillations of q_s shown in fig. 1.

An energy shift of the position of the tunneling peak caused by a magnetic field in a resonant tunneling device was observed experimentally [3, 5]. Lok *et al.* [3] pointed out that this energy shift is due to an acoustic-phonon–like excitation which appears by the extraction of the tunneling electron out of the 2DES. Our model explains this vacancy as a polarization of the 2DES due to the remote electron. When an electron tunnels from a 2DES, *i.e.* from the emitter at $z = 0$, to the quantum well at $z = d$, this electron loses an energy $W(d,B) = E(d,B) - E(0,B)$. Because neither the kinetic energy $\hbar\omega_\mathrm{c}/2$ nor the spin term depend on d, this energy difference is also represented as $W(d,B) = \delta E(d,B) - \delta E(0,B)$. As a function of the magnetic field, the energy $W(d,B)$ oscillates and strongly affects the resonant bias. When the magnetic field changes from zero to B, the resonant bias changes due this polaronic effect by an energy shift $\Delta(d,B) = W(d,B) - W(d,0)$.

In fig. 3, $\Delta(d,B)$ is plotted as a function of the external magnetic field B for the tunneling distance $d = 15$ nm with $\Gamma_0 = 1.5$ meV/T$^{1/2}$ (solid curve) and 3.0 meV/T$^{1/2}$ (dashed curve) for $n_\mathrm{s} = 3 \times 10^{11}\mathrm{cm}^{-2}$. These traces are compared to the experimental data of Lok *et al.* [3], shown as solid circles. Our results follow qualitatively the oscillation of $\Delta(d,B)$ with magnetic field, but they are about a factor of 4 smaller than the effect seen experimentally. This is possibly a consequence of neglecting correlations in the 2DES which would likely enhance Δ (see, *e.g.*, ref. [8]). In fig. 4, the energy shift $\Delta(d,B)$ is plotted as a function of the electron density n_s for various d. Note that $\Delta(d,B)$ increases with n_s as one would expect, but the increase is less than linear in n_s.

In conclusion, extending a previous theory of metallic electrons and its dielectric response, we have formulated the Coulomb correlation between a remote electron and a 2DES as a *plasmon polaron* problem. The polarization in the 2DES induced by a remote electron is regarded as a collective excitation of plasmons whose frequency is modified by the dielectric function. The energy correction due to this polaron effect is related to the experimentally observed energy shift in a tunneling experiment. Our results confirm the oscillation of the energy shift with magnetic field. Notice, moreover, that the polaron-produced oscillations are superimposed on the self-consistent adjustment of the 2DES density in experiments, where a 3D contact is effectively serving as a reservoir, and pins the Fermi energy. A direct capacitive measurement of this self-consistent shift would be valuable in a more complete analysis of the magnitudes and field dependence of the different effects. We are hopeful that this approach motivates further experiments.

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