1 複素数 Complex number

1.1

Let a complex z be $z = \cos\theta + i\sin\theta$ with a real θ. Find the expression for each of these quantities;

$$|z|, \quad \mathrm{Re}z, \quad \mathrm{Im}z, \quad \arg z,$$
$$|z^2|, \quad \mathrm{Re}z^2, \quad \mathrm{Im}\ z^2, \quad \arg z^2.$$

1.2

Prove the following identities for complex numbers α and β;
(1) $|\alpha|^2 = \alpha\bar{\alpha}$ (2) $|\bar{\alpha}| = |\alpha|$ (3) $|\alpha\beta| = |\alpha||\beta|$

1.3

Let a complex number z be defined as $z = x + iy$ using real numbers x and y.

1. Separate $z^2 = 1 + i2$ into the real and imaginary parts and derive a simultaneous equation (two algebraic equations) about the variables x and y.
2. Find all solutions of this simultaneous equation.