3　オイラーの公式 Euler's formula

3.1

Solve the following two questions:
(a) Suppose θ be a real number. Derive the following formulae;

$$\frac{1}{e^{i\theta}} = e^{-i\theta}, \qquad \overline{(e^{i\theta})} = e^{-i\theta} \qquad |e^{i\theta}| = 1.$$

(b) How should these formulae be changed if θ is a complex number $x + iy$ with real x and y?

3.2

Consider the complex number z being defined as $z = \alpha + re^{i\theta}$with a positive r, a real θ and a complex $\alpha(= a + ib)$. Draw a locus of z in Gaussian plane for the following cases:

(1)θ varies in [0,π3/2] and r is a constant,
(2)θ is a constant and r varies in [1/2,2],
(3)$r = 0.5\sin(2\theta)$ ($0 \leq \theta < 2\pi$).

3.3 p.18

Answer each of following questions.
(a) Establish the identity for any of complex number $z(\neq 1)$,

$$1 + z + z^2 + \cdots + z^{n-1} = \frac{z^n - 1}{z - 1}, \qquad (*)$$

(b) Substite $e^{i\theta}$ for z in $(*)$ and consider its imaginary parts. Then, derive the following identity:

$$\sin\theta + \sin 2\theta + \cdots \sin(n-1)\theta = \frac{\sin(n\theta/2)}{\sin(\theta/2)} \sin\frac{(n-1)\theta}{2}.$$