

## 4 複素数のべき乗と根 Powers and roots of a complex number

### 4.1

Let  $\alpha$  be a complex number defined as  $\alpha = r(\cos \theta + i \sin \theta)$  with a positive  $r$  and a real  $\theta$  ( $-\pi < \theta \leq \pi$ ). Prove the following identity for any integer  $n$ ,

$$\alpha^n = r^n [\cos(n\theta) + i \sin(n\theta)].$$

### 4.2

Locate the following values in Gaussian plane:  $(1 - i\sqrt{3})^n$  for  $n = \pm 1$  and  $\pm 3$ .

### 4.3

Assure that the  $m$ -th root of  $\alpha = re^{i\theta}$  is expressed as

$$\alpha^{1/m} = r^{1/m} \left[ \cos\left(\frac{\theta}{m} + \frac{2\pi k}{m}\right) + i \sin\left(\frac{\theta}{m} + \frac{2\pi k}{m}\right) \right].$$

with any of integer  $k (= 0, \pm 1, \pm 2, \dots)$ . Try to illustrate why  $k$  is sufficient by limiting as  $k = 0, 1, 2, \dots, m-1$ , and we have no need to consider negative  $k (< 0)$ .