

## 5 三角関数・双曲線関数 Trigonometric/Hyperbolic function

### 5.1

Prove the following identities,

$$(a) \sin iz = i \sinh z, \quad (b) \cos iz = \cosh z, \quad (c) \tan iz = i \tanh z,$$

### 5.2

Let  $z$  be  $z = x + iy$ . Prove the following identities;

- (1)  $\cos z = \cos x \cosh y - i \sin x \sinh y$ ,
- (2)  $|\cos z|^2 = \cos^2 x + \sinh^2 y$ ,
- (3)  $|\sin z|^2 = \sin^2 x + \sinh^2 y$ .

### 5.3

Let  $z$  be a complex number defined by  $z = x + iy$  with real  $x$  and  $y$ . Find all solutions that satisfy an equation  $\cos z - i = 0$  following next steps:

- (a) Find real and imaginary parts of  $\cos(z)$  as the function of  $x$  and  $y$ .
- (b) Find relations of  $x$  and  $y$  to satisfy  $\operatorname{Re}[\cos z - i] = 0$  and  $\operatorname{Im}[\cos z - i] = 0$ .
- (c) Find all solution of  $\cos z - i = 0$ .