6 複素積分1 Complex integral -patt 1-

6.1

Provided that m and n are integers, show that

$$\int_0^{2\pi} e^{im\theta}e^{-in\theta}d\theta \;=\; \begin{cases} 0 & \text{when } m \neq n \\ 2\pi & \text{when } m = n \end{cases} .$$

6.2

Let C_R denote circle $|z - z_0| = R$ taken counterclockwise with a positive real R and a centre z_0. Using the parametric representation $z = z_0 + Re^{i\theta}(0 \leq \theta < 2\pi)$ for C_R, evaluate the following integrations;

$$(a)\ \lim_{R\to 0}\int_{C_R}\frac{dz}{z^2 - z_0^2},\quad (b)\int_{C_R}\frac{dz}{z^2 - z_0^2},\ (|z_0| > R),\quad (c)\int_{C_R}\frac{dz}{z^2 - z_0^2},\ (2|z_0| < R).$$

6.3 Cauchy-Goursat integral formula

Let C denote the positively oriented circle $|z| = 2$. Evaluate the following integrals:

$$\text{(a)}\quad \int_C \frac{e^z}{z+1}dz, \qquad \text{(b)}\ \int_C \frac{e^z}{(z+1)^2}dz, \qquad \text{(c)}\ \int_C \frac{e^z}{(z^2+1)^2}dz.$$