7　複素積分 2 Complex integral　-part 2 -

7.1

Let $f(z)$ be a regular function, and let S_R be a semicirclr with radius R in the upper Gaussian plane. Prove the following *triangle inequality*:

$$\left|\int_{S_R} f(z)dz\right| \leq \int_{S_R} |f(z)| \cdot |dz|.$$

Then, evaluate the supremum of the limits below;

$$\text{(a)} \qquad \lim_{R\to\infty}\left|\int_{S_R}\frac{dz}{z^2+1}\right|, \qquad\qquad \text{(b)} \qquad \lim_{R\to\infty}\left|\int_{S_R}\frac{zdz}{z^2+1}\right|.$$

7.2

Find integral $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{x^2+1}$ using complex integral $\displaystyle\int_C\frac{dz}{z^2+1}$ with an apropriate path C in Gaussian plane.

7.3

To find the integral $I = \displaystyle\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)d\theta$, follow the speps below;

(a) Evaluate the integral $I_{ex} = \displaystyle\int_C \frac{e^z}{z}dz$ about C: $z = e^{i\theta}$ of $0 < \theta < 2\pi$.

(b) Show that $I = \mathrm{Im}\big[I_{ex}\big]$.