2 変数分離形 Variable separable equation

2.1 p.9

Consider the following differential equation,

$$x(x-1)\frac{dy}{dx} + y = 0. \tag{$*$}$$

(a) Find the general solution of $(*)$.

(b) Eliminate an arbitrary constant in the solution obtained in (a) and verify whether $(*)$ can be derived.

(c) Verify whether the solution derived in (a) satisfys the equation $(*)$.

2.2 11

Concerning with each of the following differential equations, find the *particular solution* which satisfies the condition enclosed in the brackets;

(1) $\sqrt{x}y' = \sqrt{y+1} \quad (x = 4, y = -1)$

(2) $y' = 2x(1+y^2) \quad (x = 0, y = 0)$

(3) $(1+x^2)y' = \sqrt{1-y^2} \quad (x = 1, y = 1)$

Here, the dash $'$ means the differential with respect to x.

2.3

Let $f(y)$ be a general function of y, and the function $y(x)$ is differentiable, then,

$$\int_{x_0}^{x_1} f(y(x))\frac{dy}{dx}dx = \int_{y_0}^{y_1} f(y)dy, \tag{$**$}$$

provided that the function $y = y(x)$ satisties $y_0 = y(x_0)$ and $y_1 = y(x_1)$ at the ends of integration interval.

Use the relation $(**)$ and find the *particular solution* of $\sqrt{x}y' = \sqrt{y+1}$ satisfying the the initial condition $y = 1$ at $x = 4$.