4　1階の線形微分方程式

Linear differential equation of first order

4.1 p.23

Consider the following differential equation,

$$x\frac{dy}{dx} + y = xe^x. \qquad (*)$$

(a) Find the general solution of $(*)$.

(b) Eliminate an arbitrary constant in the solution obtained in (a), and verify whether $(*)$ can be derived.

(c) Verify whether the solution derived in (a) satisfys the equation $(*)$.

Useful Formulae$^{p.18}$

Let P and Q be an arbitrary function of x. Then, The following relations can be established;

$$(y' + Py)e^{\int P dx} = \frac{d}{dx}\left(ye^{\int P dx}\right), \qquad y = e^{-\int P dx}\left[\int Qe^{\int P dx}dx + C\right].$$

4.2 p.20

Find the general solution for each of these equations;

(1) $xy' + y = x^2$, (2) $(1+x^2)y' = xy + \sqrt{(1+x^2)^3}$.

4.3 p.19,p.23

Find the general solution for each of these equations;

(1) $y' + y = \cos(px)$, (2) $y' - y\tan x = \cos x$.

Here, p is real and positive.