5 完全微分方程式

Exact differential equation

5.1 Exact Differential Form

Derive following values of the exact differentials:

(1) $d\Big(\dfrac{y}{x}\Big)$, (2) $d\sqrt{x^2+y^2}$, (3) $d\left(\dfrac{x-y}{x+y}\right)$.

5.2 p.27, 33

Examine whether each of the following differential equations is exact. If it is exact, find its general solution;

(1) $(3x+y)dx+(x-2y)dy=0$ (2)$(x-y)dx+(\frac{1}{y^2}-x)dy=0$

(3) $(x^2+3xy)dx+(3x^2-xy)dy=0$,

Useful formula$^{pp.23-25}$

Consider an exact differential equation expressed as

$$P(x,y)dx+Q(x,y)dy=0, \qquad (*)$$

with the assumption that $\frac{\partial P}{\partial y}=\frac{\partial Q}{\partial x}$. Then, the solution of $(*)$ can be expressed as

$$\int_a^x P(x,y)dx+\int_b^y Q(a,y)dy=0.$$

5.3 p. 33

Find the *particular solution* for the following exact differential equation,

$$(y^2e^x+\sin y)dx+(2ye^x+x\cos y)dy=0,$$

satisfying the initial condition $y=0$ at $x=0$.