

8 ベキ級数による解 Solutions expressed in power series

8.1 p.102

Assume that the two functions $f(x)$ and $g(x)$ can be expressed in series as

$$\begin{aligned}f(x) &= a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots, \\g(x) &= b_0 + b_1x + b_2x^2 + \cdots + b_nx^n + \cdots,\end{aligned}$$

with the coefficients a_n and b_n ($n = 0, 1, 2, \dots$). Find the coefficients of power series for the product $f(x)g(x)$.

8.2 p.106

Find the solution expressed in power series of x for each of the following differential equations;

$$(a) \quad y' = x + 2xy, \qquad (b) \quad (x + 1)y' - y = x(x + 1).$$

8.3 Bessel function

Find a solution of the differential equation as follows,

$$x^2y'' + xy' + x^2y = 0,$$

by expressing y in power series of x . (No need to find two solutions.)